Estimating the Diagnostic Yields Resulting from Renography and Deconvolution Parameters: A Logistic Regression Analysis

V. Kempi and D.G. Sutton

Department of Radiophysics, Sjukhuset, Ostersund, Sweden and Medical Physics Department, Ninewells Hospital and Medical School, Dundee, Scotland

Methods: Seventy patients with established diagnoses of normal, parenchymally insufficient or acutely obstructed kidneys were subjected to gamma camera renography. Deconvolution was then performed using three main techniques subdivided into six variants. Parameters from time-activity curves as well as retention curves were calculated. Logistic regression analysis was performed to assess the ability of renography and deconvolution methods to differentiate between kidney groups. **Results:** Discrimination between the groups was achieved by standard renography using six of 17 tested renogram parameters. Based on a set of six curve parameters, the correct classification rates ranged 86%–100%. Five of the six variants of the deconvolution technique used produced similar results. None, however, produced results which were as robust as those from renography. The sixth deconvolution method was consistently worse than the others. **Conclusion:** Standard renography was consistently better than any of the deconvolution techniques used in the separation of the kidney groups. Conceptually, the results of a logistic regression analysis of renogram parameters may raise possibilities in the field of computer-aided diagnosis.

Key Words: renography; deconvolution; logistic regression; classification

J Nucl Med 1995; 36:147–152

As part of the data analysis resulting from renography, a number of parameters can be extracted from time-activity curves and the retention functions resulting from deconvolution. Renogram parameters have been shown to facilitate the assessment of the curves (*1–4*) and therefore can help in renal function classification. Parameters from deconvolution have also been shown to be useful (*5,6*). For a successful classification of kidney function it is of obvious importance to know what method to use, which parameters contribute to the separation of the renal conditions and which do not.

Received Nov. 29, 1993; revision accepted Sept. 13, 1994.

For correspondence or reprints contact: V. Kempi, PhD, MD, Dept. of Radiophysics, Sjukhuset, S-831 83 Ostersund, Sweden.

Recently, it was shown that renogram parameters can be used to objectively separate kidneys with dysfunction from those with normal function with an overall accuracy of 86% (*7*). In that study, the kidneys were divided into two broad groups: normal and pathological. In the present study, the kidneys previously defined as being pathological have been further subclassified into two groups: those with parenchymal insufficiency and those with acute outflow obstruction. This classification strategy allows an assessment to be made of the ability of standard renography and deconvolution to discriminate between normal functioning kidneys and different types of kidney dysfunction. Moreover, in the interest of completeness, a number of curve parameters not considered in the previous study (*7*) have been included in this analysis.

The aim of the present study is to extend the previous analysis and thereby determine which analysis technique, when used with a range of renal pathology, provides the most consistent results in clinical practice. Implicit in this aim is the identification of those subsets of parameters which are good predictors of the absence or presence of renal dysfunction. A subsidiary aim is to study the ability of renography combined with each deconvolution method in turn to distinguish among the kidney groups.

PATIENTS AND METHODS

Seventy consecutive patients were studied (35 women, age range 16 to 74 yr, median 45; 35 men age range 21 to 85 yr, median 53). Six patients had had a nephrectomy 4 yr or more before the examination. One kidney with virtually no function was omitted from the analysis. The patients were studied as a matter of routine and were requested mainly from the department of internal medicine, the department of surgery or the outpatient clinics. Among the aims expressed on the request forms were renal function determination in subjects with hematuria, proteinuria, urinary tract infection, hypertension or pain suggestive of renal involvement. In other cases, renography was requested to decide whether or not kidneys were ureterically obstructed, to confirm release of stones or to study the recovery or deterioration of renal function during treatment.

All patients had established renal conditions (*8,9*). The final diagnoses were based on radiological investigation (IVU and/or

KUB), biochemical tests (serum creatinine, serum urea and test tapes sensitive to albumin, haemoglobin and glucose), urinary microscopy and clinical follow up. The final diagnosis was made on the basis of follow up over periods ranging from several months (in the normal group) to several years. The patients in the normal group were found to have transient haematuria or transient proteinuria or transient pain suggestive of ureteral stone.

There were three main groups: one group of 81 normal kidneys, a second group comprising 30 kidneys with parenchymal insufficiency and a third group of 22 kidneys with acute outflow obstruction.

Since it can easily be shown that a subject's two kidneys demonstrate some degree of correlation (*8*), calculations were based on one kidney per patient. The kidney to be analyzed was randomly chosen, except in the seven patients with one functioning kidney, with the result that 43 normal kidneys, 18 kidneys with parenchymal insufficiency, and nine kidneys with acute outflow obstruction were entered into the study.

Scintillation camera imaging was performed with 80 MBq of ^{99m}Tc-DTPA. The camera had a 25 cm diameter NaI(Tl)-crystal which was 12.5 mm thick and imaging was performed with a 1200 hole medium-energy collimator. Energy discrimination was carried out with 20% window centred over the 140 keV photopeak. The camera was interfaced to a nuclear medicine computer system. The equipment used was the same for all patients. One hour before the imaging procedure, the patient was hydrated orally with 10 ml water per kg body weight. In all cases the activity was measured prior to injection into an antecubital vein and was found to range between 78 and 82 MBq. The volume of activity administered was less than 0.5 ml and the bolus injection technique used ensured that there was virtually no residual activity left in the syringe. In no case was there evidence of any significant extravasation of the injection.

Sequential 8-bit deep 64 by 64 scintigrams, each of 20 sec duration were recorded for a total of 15 min. The methodology for the selection of the regions of interest (ROIs) has previously been described (*9*) but is briefly reiterated for ease of reference. One ROI was placed over each kidney and one between them to represent blood background. Where possible, irregular ROIs defining renal parenchyma alone were chosen. When the pelvis could not be adequately visualized, rectangular ROIs were placed over the entire kidney. The blood background ROI was defined by a rectangle, the height of which was identical to that of the largest kidney and the width of which was maximized making sure that the ureters, the pelves and the urinary bladder were excluded. The blood ROI was normalized to a constant size to standardize the amplitude of the retention function in order to make direct comparisons possible (*10*).

Before calculating curve parameters in conventional renography and deconvolution, the renal time-activity curves (TAC) were corrected by normalized background-subtraction which is a simple and straightforward technique, and probably still the most common in renography. A vascular spike (*10,11*) on the TAC, frequently observed at 30 sec after injection, i.e., the second curve point, was removed by bounding (*10*). The TAC were then subjected to three smooths using the smoothing operator (*9,12,17*). Prior to deconvolution, the data points of the first frame were excluded from calculations (*10*).

Renogram Parameters

The renogram parameters derived from the TAC are summarised in Table 1. Three count-based parameters were used, the

TABLE 1
Renogram and Deconvolution Parameters

Parameter	Definition
C_{110}	Countrate at 110 sec postinjection
C_{Max}	Countrate at peak activity of renogram curve
Uptake ratio	C_{110} divided by countrate at 50 sec postinjection
AUC	Area under renogram curve (total counts)
ROU	Rate of uptake between 50 and 110 sec postinjection
RROUD	ROU divided by the curve's C_{Max}
ARROU	ROU divided by the C_{Max} of the upper curve
FRROU	ROU divided by the curve's C_{110}
AFFROU	ROU divided by the C_{110} of the upper curve
T_{Max}	Time to peak activity in renogram curve
T_{ROD}	Time to maximum downslope of renogram curve
ROD	Rate of decrease at T_{ROD}
RROD	ROD divided by the curves's C_{Max}
ARROD	ROD divided by the C_{Max} of the upper curve
FRROD	ROD divided by the curve's C_{110}
AFRROD	ROD divided by C_{110} of the upper curve
ER	Peak activity divided by activity at 890 sec
Amp	Amplitude of renal retention curve
MTT	Mean transit time of retention curve
MaxTT	Maximum transit time of retention curve

count rate at 110 sec postinjection (C_{110}), at peak activity (C_{Max}), and the area under the renogram curve (AUC) which was defined as the total number of counts. T_{Max} was defined as the time to peak activity and the rate of uptake (ROU) as the difference between the count rate at 110 sec and at 50 sec postinjection, divided by the difference in time (60 sec). Four variants of relative rate of uptake were defined. They were based upon normalizing ROU in four different ways. Normalization was performed by the curve's peak activity (RROU) or by that of the upper curve (ARROU), by the count rate at 110 sec postinjection (FRROU) or by that of the upper curve (AFRROU), respectively. For each patient the upper curve was defined as being the renogram with the greatest peak activity. In the case where the upper curve represented the kidney analyzed, RROU and ARROU were identical as were FRROU and AFRROU. The uptake ratio was defined as the count rate at 110 sec, divided by that at 50 sec. In the excretion phase of the renogram the time to maximal downslope (T_{ROD}) was determined as was the maximum rate of decrease (ROD). ROD was defined as the difference between the count rate at time T_{ROD}-30 sec and at time T_{ROD}+30 sec divided by the difference in time (60 sec). Four variants of relative rate of decrease were defined based on the normalization of ROD. Normalization was performed by the curve's peak activity (RROD) or by that of the upper curve (ARROD), by the curve's count rate at 110 sec postinjection (FRROD) or by that of the upper curve (AFRROD), respectively. In the case where the upper curve represented the kidney analyzed, RROD and ARROD were identical as were FRROD and AFRROD. The slope parameters ROU, ROD, and their extensions were all based on least-squares calculations. ARROU, AFRROU, ARROD, and AFRROD were designed to take into account asymmetries of renal function. Finally, the excretion ratio (ER) was defined as the peak activity divided by the activity at 890 sec. Of the renogram parameters C_{110}, C_{Max}, AUC, ROU, and ROD are activity dependent.

Deconvolution Techniques and Parameters

Deconvolution was performed using variants of three main techniques. The first technique was based on constrained least-squares restoration (*7,12*) with (CLSRP) and without (CLSR) an initial plateau, and the second technique was matrix algorithm method (MA) and its alternatives AMA and AMA$^+$. The third main technique (FFTC) was provided on commercial software. The FFTC method was based on direct Fourier transform division, with a scaled cosine appended to avoid problems with an abrupt time window (*13,14*). The alternative matrix method AMA has been used by Carlsen (written communication, 1987) but its derivation has never been published. A derivation of the AMA method is therefore given in the Appendix.

The deconvolution parameters evaluated in the present study are summarised in Table 1. They were amplitude of the retention function (Amp), mean transit time (MTT), and maximum transit time (MaxTT). The amplitude resulting from the AMA technique is forced to 1.00 and therefore can not contribute to any model. To address this problem, a variant of the AMA technique (AMA$^+$) in which the AMA parameters were combined with the amplitude from the matrix algorithm was also entered to the study. In all cases, the MaxTT was defined as the time at which the retention function crossed the time-axis and the MTT as the area under the retention curve, divided by its amplitude (*10,12*).

Statistical Analysis

Statistical tests were performed with logistic regression based on forward stepwise variable selection. Before logistic regression, however, all variables were checked to ensure linearity in the logit (*15*). Tests were then made for confounding and interactions in the model. For example, since it is known that values of GFR are age dependent (*16,17*), confounding and/or interaction by age are of special interest when analysing renogram parameters. The tests showed that the variable age was a potential confounder. Therefore, the age variable was forced into each logistic regression model and prevented from being removed. Tests on interaction by age, on the other hand, did not prove any interaction of significance.

The score statistic was used for entering variables into the model whereas the likelihood-ratio test determined variables to be removed from the model. The default significance level for entry was <0.05 while that for removal was 0.10. The significance levels for entry and removal and the goodness-of-fit statistics determined when to halt the regression analysis. The order of entry of the parameters gives their relative importance to the separation of the kidney groups studied.

The logistic regression analysis determined the ability of conventional renography and each deconvolution method to distinguish between the various kidney groups. In addition, a combined model using a combination of all renogram and deconvolution parameters was generated. In each analysis the percentage of correctly classified kidneys, i.e., the overall accuracy, was obtained. The classification results ranked the techniques. In the case of tied results in the individual models, the technique having parameters with the greatest significance in the combined model received the best rank.

Models with numerical problems as manifested by a large estimated standard error of an estimated logistic regression coefficient relative to the point estimate (*15*) were not accepted as final models. When encountered, the removal or inclusion of a variable generally cured the model during the continuous model building. In none of the models used did the resulting significance levels for the models ($P > 0.15$) differ from a perfect model.

The statistical analysis including logistic regression was performed by commercially available software (*18*) implemented on a personal computer.

TABLE 2
Results of Initial Subjective Interpretation of Conventional Renograms

		Final diagnosis		
		Normal	Parenchymal insufficiency	Acute outflow obstruction
Subjective	Normal	40	5	2
	Parenchymal insufficiency	2	11	0
Interpretation	Acute outflow obstruction	1	2	7

Overall accuracy is 83% in 58 of 70 kidneys.

RESULTS

Table 2 presents the results of the initial subjective interpretation of the conventional renograms, performed at the time of the studies and before the final diagnosis was determined. Apart from sequential scintigrams of the kidneys and the curve-patterns, the information available at the time of the studies was age, sex, clinical data in the request forms and the renogram parameters: ROU, T_{Max}, C_{Max}, and ER as defined above. The overall accuracy of the subjective interpretation was 83%. In a previous study (7) it was taken as being 86% since the group of kidneys with pathological function was not further subclassified into kidneys with parenchymal insufficiency and acute outflow obstruction. In fact, in the original subjective interpretation two kidneys with insufficiency were erroneously considered to be obstructed.

Table 3 lists the curve parameters which contributed to the separation of the renal groups as obtained with renography and each deconvolution technique. Parameters not shown did not contribute. The correct classification rate, i.e., the overall accuracy in each analysis is shown in Table 4 and the ranks of the four best differentiating techniques are shown in Table 5 and described in the following paragraphs. In all cases, a combined model produced no improvement over the best single model obtained, which was invariably produced using renogram parameters.

Kidneys with Normal Function Versus Those with Pathological Function

Sixty of the 70 (86%) kidneys were correctly classified by conventional renography when the diagnoses (normal and pathological renal function) were used as the grouping variable. The variables accepted for inclusion into the model were ER, C_{110} and T_{Max}, in that order.

The best deconvolution technique was CLSR or CLSRP which were inseparable whereas the second best was AMA$^+$ as shown in Table 5. They produced correct classifications of 83% and 81%, respectively. The principal

TABLE 3
Predicting Curve Parameters in Renography and Deconvolution

Kidney groups	Renography	MA	AMA	AMA⁺	CLSR	CLSRP	FFTC
		Deconvolution method					
N vs. IO	ER C_{110} T_{Max}	MaxTT Amp	MaxTT	MaxTT Amp MTT	MaxTT	MaxTT	MTT
N vs. O	T_{Max} T_{ROD}	MaxTT	MaxTT MTT	MaxTT MTT	MaxTT	MTT	MaxTT
N vs. I	C_{Max} AUC	Amp	—	Amp	Amp	Amp	MTT
O vs. I	C_{Max}	Amp MTT	MaxTT	Amp MaxTT	—	—	—
O vs. NI	AUC C_{110}	MaxTT	MaxTT	MaxTT	MTT	MTT	—
I vs. NO	C_{Max} AUC	Amp MTT	—	Amp	Amp	Amp	MTT

N = normal; I = parenchymal insufficiency; O = outflow obstruction; MA = matrix algorithm; AMA = alternative matrix algorithm; AMA^+ = AMA with amplitude from matrix algorithm method included; CLSR = constrained least-squares restoration; CLSRP = CLSR with initial plateau; FFTC = commercial software based on FFT.

predictor was the MaxTT. For the remaining deconvolution techniques the classifications ranged 70%–80%.

Normal Kidneys Versus Those with Ureteral Obstruction

The renogram parameters T_{Max} and T_{ROD} were accepted for inclusion and resulted in complete separation of the two renal groups, i.e., the correct classification rate was 100% as seen in Table 3 and 4, respectively. Of the deconvolution techniques, AMA (or AMA^+) was the method of first rank. The parameters MaxTT and MTT resulted in a classification rate of 92%. Since the amplitude parameter from the matrix method did not meet entry criteria, the results of the methods AMA and AMA^+ were identical.

TABLE 4
Classification Results in Renography and Deconvolution

Kidney groups	No. of kidneys	Renography	MA	AMA	AMA⁺	CLSR	CLSRP	FFTC
			Deconvolution technique					
N vs. IO	43 vs. 27	86	80	71	81	83	83	70
N vs. O	43 vs. 9	100	90	92	92	90	90	87
N vs. I	43 vs. 18	89	84	—	84	79	79	74
O vs. I	9 vs. 18	100	96	85	93	—	—	—
O vs. NI	9 vs. 61	97	90	89	89	90	90	—
I vs. NO	18 vs. 52	90	86	—	87	86	86	81

Figures are percentages of correct predictions, that is, overall accuracy.

TABLE 5
Ranking of Renography and Deconvolution Techniques

Kidney groups	Rank 1	2	3	4
N vs. IO	Renography	CLSR or CLSRP	AMA†	MA
N vs. O	Renography	AMA or AMA⁺	CLSRP	CLSR
N vs. I	Renography	MA or AMA⁺	CLSR	CLSRP
O vs. I	Renography	MA	AMA⁺	AMA
O vs. NI	Renography	MA	CLSRP	CLSR
I vs. NO	Renography	MA or AMA⁺	CLSRP	CLSR

Normal Kidneys Versus Those with Insufficiency

Use of renogram parameters resulted in an 89% correct classification rate based on the parameters C_{Max} and AUC. Among the deconvolution techniques, the matrix algorithm was the method of the first rank, based on the sole parameter Amp, resulting in an overall accuracy of 84%. Since only the Amp parameter was eligible for inclusion, the AMA method failed to distinguish between the kidney groups. The remaining techniques had classifications ranging 74%–79%.

Kidneys with Ureteral Obstruction Versus Those with Insufficiency

In renography the sole variable eligible for inclusion was C_{Max} which completely separated the two kidney groups, i.e., the classification was 100%. The matrix algorithm technique, based on the parameters Amp and MTT, was the deconvolution method of first rank and had a classification of 96%. The CLSR, CLSRP and the FFTC techniques all failed in producing useful parameters.

Ureteral Obstruction Versus Normal and Insufficiency

Based on the variable AUC and C_{110}, renography correctly predicted 97% of the kidneys. The matrix algorithm was the deconvolution method of the first rank and as such it produced a correct classification of 90% with MaxTT as the only predicting parameter. The classification was the same, i.e., 90% in case of CLSRP and CLSR with the parameter MTT. Consequently, ranking was based on significance levels in the combined model significance. The FFTC technique failed in producing differentiating parameters.

Insufficiency Versus Normal and Ureteral Obstruction

Based on the parameters C_{Max} and AUC, renography correctly classified 90% of the kidneys. The deconvolution method of the first rank was MA (or AMA^+ using the Amp parameter from the MA technique). The remaining deconvolution techniques had classifications ranging 81%–86%.

DISCUSSION

Renal function parameters are not necessary to tell us whether or not a patient has a renal dysfunction. However, quantitative methods of analysis utilizing objective data are usually considered to be superior to subjective interpretation. In this study we have used a multivariate statistical method to compare the results of several types of objective analysis. In multivariate statistical procedures the emphasis is on analyzing variables together (i.e., renal function parameters), not one at a time. By considering the parameters simultaneously, we were able to identify those parameters useful in making predictions. Of course, even when objective analysis is performed, complementary information is obtained by examining the renograms since, for example, one of the kidneys may often serve as a standard with which the other kidney is compared.

In the case of renography six of 17 available parameters (AUC, C_{110}, C_{Max}, T_{Max}, T_{ROD} and ER) contributed to the differentiation of the kidney groups studied. In one of the tests of renography three parameters specified the model, in another just one parameter sufficed, and in the remaining four tests two parameters were found to be adequate. All six parameters are easily obtained and four of them (C_{110}, C_{Max}, T_{Max}, T_{ROD}) could be derived directly from time-activity curves. It is fortunate that the prediction of the various kidney groups was achieved by a small number of parameters. The fewer the number of parameters the better, since such models are more likely to be numerically stable and are more easily generalized (*15*). In the present study no slope parameters were accepted for inclusion into the logistic regression models. The C_{110} parameter substituted the ROU parameter which previously (*7*) was part of the model. This finding does not imply that slope parameters did not contribute to the separation of the kidney groups. The explanation is that the significance of the slope parameters were not sufficient when highly differentiating parameters such as those seen in Table 3 were taken into consideration.

Interestingly, the usefulness of the parameters T_{Max} (*19–21*) and C_{Max} (*21*) was suggested some 30 years ago. The choice of renogram parameters could probably be improved. For instance, the area under the curve was not optimized. Instead, for simplicity, it was defined over the entire length of each study. To distinguish between the kidney groups, a shorter time interval could be more appropriate.

Perhaps it deserves mentioning that the selection of curve parameters is probably affected by the renal conditions studied as well as the hydration state of the patients. Some of the renogram parameters will depend on the glomerular filtration rate and/or the renal transit time. In another set of renal conditions or in another hydration state other sets of parameters than those shown in Table 3 could prove more useful. The same holds true for subjects premedicated with frusemide or captopril etc.

· The ability of each deconvolution method to distinguish among the kidney groups was less efficient than that of renography. No one deconvolution method consistently produced the most robust results although the FFTC technique did produce the most consistently bad results. The deconvolution parameters amplitude and MaxTT were important in the case of the matrix algorithm technique and its alternative AMA^+. Only in the case of CLSRP was the MTT parameter of primary importance (see Table 3). This emphasises the usefulness of amplitude and MaxTT as deconvolution parameters in a clinical environment (*8*). It is evident that the five viable deconvolution methods produce results which have meaning in a clinical environment. This is of some importance since there are situations when deconvolution provides help and conventional renography does not. For instance, in patients with extravascular injections deconvolution corrects for the poor blood input function.

It was observed that both the CLSR and CLSRP techniques failed to discriminate between obstructed and insufficient kidneys. However, a slight change in the entry acceptance level from $p < 0.05$ to $p < 0.08$ resulted in successful discrimination with 96% overall accuracy. The implication is that the CLSR techniques did not fail per se, but in fact did not produce models which were sufficiently robust for the stringent strategy adopted here. It is perhaps worth emphasising that such a stringent regime was adopted to ensure that a sharply defined split between groups was produced. This approach differs from some other uses of logistic regression where the aim is the identification of risk factors and variable rich models are perhaps preferable.

The scanty outcome with commercial software is hard to explain since there is no access to the source code.

To test the possibility of using the combined information provided by conventional renography and a deconvolution technique, it seemed appropriate to look for a combined model that produced better results than those obtained with renography alone. Such a model, however, could not be produced. In those situations where a deconvolution parameter met entry criteria, it did not significantly improve the fit of the model nor the classification. For routine use it is, of course, unrealistic to utilize all information provided by parameters from a number of deconvolution methods combined with those from renography.

The kidneys with dysfunction did not form homogeneous groups in the sense that those with reduced renal function had a varying degree of parenchymal insufficiency and those with acute outflow obstruction had a varying degree of obstruction none of which was totally obstructed. Such varying degrees are, of course, expected in consecutive series of patients. The wide range regarding the degree of insufficiency and obstruction inevitably makes a separation of the various kidney groups more complicated, especially when one of the groups consisted of both obstructed kidneys and kidneys with parenchymal insufficiency. This contributed to the limited classifications seen in Table 4 in the analyses of kidneys with normal function versus those with dysfunction due to parenchymal insufficiency and outflow obstruction.

It is interesting to speculate that since the classification resulting from the renogram parameters was high, they may be used as a basis for the development of a systematised tool to aid in the diagnosis of renograms. Conceptually, this might be done by taking the coefficients resulting from the logistic regression model and using them to generate a notional probability of disease for a particular set of patient data. Such an undertaking would require considerable validation. Future work is needed to further subdivide the classification between pathological kidneys of various diseases. For instance, a potentially profitable avenue in future work is to continue with an investigation of the usefulness of the methods and parameters used here in subjects with renal artery stenosis (*22*). In a recent editorial, emphasis was placed on the fact that it remains to be determined which is the best way to interpret these studies with special reference to captopril renography (*23*). In the same editorial the question was posed as to whether any one quantitative parameter was better than others (*23*). The present study, although in an other clinical environment, has at least to some extent responded to such questions.

In conclusion, standard renography, based on six of 17 tested renogram parameters, was consistently better than any of the deconvolution techniques used in the separation of the kidney groups. The results from five of the six deconvolution variants were very similar while one method consistently produced poor models or failed. Conceptually, the results of a logistic regression analysis of renogram parameters may raise possibilities in the field of computer aided diagnosis.

ACKNOWLEDGMENTS

The authors acknowledge the assistance of Jämtlands läns landsting, Sweden and of British Council, Stockholm, Sweden. We would also like to thank Drs. K. Lindberger and B. Ritter, Internal Medicine Dept, Sjukhuset, Ostersund. Thanks also to Joan Campbell, Milwaukee, WI. Finally, thanks to Ove Carlsen, Vejle Hospital, Denmark for providing details of the alternative matrix algorithm.

APPENDIX

The alternative matrix algorithm method (AMA) employs split time intervals and can be explained as follows: At time zero the renal count rate, R_1 originates from the bolus, that is:

$$R_1 = R_1 . H_1,$$

where H_1 represents the very first retention value and which thus is 1.00. At time Δt later we obtain:

$$R_2 = R_1 . H_2 + (0.5 \cdot B_1 . H_2 + 0.5 \cdot B_2 . H_1) \cdot \Delta t,$$

where B_1 and B_2 represent renal input for the first and second time interval, respectively. After successive Δt intervals, the equation becomes:

$$R_3 = R_1 . H_3 + (0.5 \cdot B_1 . H_3 + B_2 . H_2 + 0.5 \cdot B_3 . H_1) \cdot \Delta t,$$

•

•

$$R_n = R_1 . H_n + (0.5 \cdot B_1 . H_n + B_2 . H_{n-1} + \cdots + B_{n-1} . H_2 + 0.5 \cdot B_n . H_1) \cdot \Delta t.$$

The weighting factor 0.5 for the first and last term in the parenthesis arises because the renal input at time zero, i.e., during the time interval $\Delta t/2$ before time zero and $\Delta t/2$ after time zero, is B_1. However, the contribution before time zero does not exist so B_1 is halved. A similar argument applies to the last term in the equation. Consequently, at time t the renal input is B_t. From B_t there is, however, a contribution only during the time interval $\Delta t/2$ to the left of time t. The remaining part of B_t, during $\Delta t/2$ to the right of time t, has not entered the kidney yet.

Using the equations above, H_i (i=1,n) is easily calculated. In practice, a constraint is needed to force H_2 to become 1.00 for each kidney. The constraint is achieved by calculating a weighting factor, applied to the blood input function for each kidney, respectively.

REFERENCES

1. Brown FA, Gelber RH, Youkeles LH, Bennett LR. Quantitative approach to the I^{131} renogram. *JAMA* 1963;186:131–134.
2. Wax SH, McDonald DF. A quantitative analysis of the ^{131}I sodium o-iodohippurate renogram in hypertensive patients. *J Urol* 1964;92:409–415.
3. Denneberg T. Critical analysis of the renogram tracing in normal subjects and in the diseased state. In: Timmermanns L, Merchie G, eds. *Radioisotopes in the diagnosis of diseases of the kidneys and the urinary tract.* Amsterdam: Excerpta Medica Foundation; 1969:222–227.
4. DeGrazia JA, Scheibe PO, Jackson PE, et al. Clinical applications of a kinetic model of hippurate distribution and renal clearance. *J Nucl Med* 1974;15:102–114.
5. Diffey BL, Hall FM, Corfield JR. The ^{99m}Tc-DTPA dynamic renal scan with deconvolution analysis. *J Nucl Med* 1976;17:352–355.
6. Britton KE, Maisey MN. Renal radionuclide studies. In: Maisey MN, Britton KE, Gilday DL, eds. *Clinical nuclear medicine.* 1st edition. London: Chapman & Hall; 1983:93–98.
7. Sutton DG, Kempi V. Constrained least-squares restoration and renogram deconvolution: a comparison with other techniques. *Phys Med Biol* 1993; 38:1043–1050.
8. Kempi V. Deconvolution analysis of renograms obtained with simultaneously administered ^{99m}Tc-DTPA and ^{131}I-hippuran. *Nuklearmedizin* 1988;27:188–194.
9. Kempi V, Persson BRR. Evaluation of renal function parameters with simultaneously administered ^{99m}Tc-DTPA and ^{131}I-Hippuran. *Eur J Nucl Med* 1983;8:65–71.
10. Kempi V. A FORTRAN program for deconvolution analysis using the matrix algorithm method with special reference to renography. *Comput Methods Progr Biomed* 1987;24:107–116.
11. Taplin GV, Meredith Jr OM, Kade H, Winter CC. Radioisotope renogram. External test for individual kidney function and upper urinary tract patency. *J Lab Clin Med* 1956;48:886–901.
12. Sutton DG, Kempi V. Constrained least-squares restoration and renogram deconvolution: a comparison by simulation. *Phys Med Biol* 1992;37:53–68.
13. Bergland GD. A guided tour of the fast Fourier transform. *IEEE Spectrum* 1969;6:41–52.
14. Juni JE, Thrall JH, Froelich JW, et al. The appended curve technique for deconvolution analysis—method and validation. *Eur J Nucl Med* 1988;14: 403–407.
15. Hosmer DW Jr, Lemeshow S. *Applied logistic regression.* 1st edition. New York: John Wiley; 1989.
16. Bianchi C. Measurement of the glomerular filtration rate. *Progr Nucl Med* 1972; 2:21–53.
17. Slack TK, Wilson DM. Normal renal function. C_{IN} and C_{PAH} in healthy donors before and after nephrectomy. *Mayo Clin Proc* 1976;51:296–300.
18. Norusis MJ. Logistic regression analysis. SPSS/PC+ advanced statistics™ 5.0, Chicago:SPSS Inc, 1992;1–33.
19. Stewart BH, Haynie TP, Arbor A. Critical appraisal of the renogram in renal vascular disease. *JAMA* 1962;12:454–459.
20. Tauxe WN, Hunt JC, Burbank MK. The radioisotope renogram (ortho-iodo-hippurate-I^{131}). *Am J Clin Pathol* 1962;37:567–578.
21. Pircher FJ, Carr EA, Patno ME. Evaluation of quantitative aspects of the radioisotope renogram. *J Nucl Med* 1963;4:117–131.
22. Fine EJ, Sarkar S. Differential diagnosis and management of renovascular hypertension through nuclear medicine techniques. *Semin Nucl Med* 1989; 19:101–115.
23. Blaufox DM. Should the role of captopril renography extend to the evaluation of chronic renal disease? [Editorial]. *J Nucl Med* 1994;35:254–256.